Math 2050, HW 2 (Due: Due: 7 Oct 2022)

(1) If $b > a > 0$, show that the sequence $x_n = (a^n + b^n)^{1/n}$ is convergent with limit equal to b.

(2) If $\{x_n\}_{n=1}^{\infty}$ is a sequence of positive real number such that $\lim_{n\to+\infty} x_n^{1/n} = L < 1$, show that x_n converges to 0 as $n \to +\infty$.

(3) (a) Let $x_1 = 10$ and $x_{n+1} = x_n + \frac{2}{x_n}$ for all $n \in \mathbb{N}$. Determine whether $\{x_n\}$ is convergent or not.

(b) Let $x_1 = 1$ and $x_{n+1} = \sqrt{4 + x_n}$ for all $n \in \mathbb{N}$, show that $\{x_n\}$ is convergent. Find the limit.

(4) Show that if the sequence $\{x_n\}_{n=1}^{\infty}$ satisfies $x_n \geq 0$ for all n and $\{(-1)^n x_n\}_{n=1}^{\infty}$ is convergent, then $\{x_n\}_{n=1}^{\infty}$ is convergent. Is the conclusion still true if we only assume $x_n \geq -1$?

(5) If $\{x_n\}_{n=1}^{\infty}$ is a sequence which is bounded from above. Let $s = \sup\{x_n\}$. Show that either $s = x_N$ for some N or there is a subsequence x_{n_k} so that $x_{n_k} \to s$ as $k \to +\infty$.